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Physics II
ISI B.Math I year.
II Semestral ~~Final~~ Exam : May 2, 2002
9.45–12.45

Instructor : Sukanya Sinha

Answer question 1 and 2 and any 5 out of the rest. All questions carry equal marks. Total Marks: 70.

1. For each of the following statements indicate whether it is true or false:

a) For a closed system the value of $\int PdV$ for the change of a gas from one given state to another is a constant regardless of the path, as long as all processes are reversible.

b)The entropy of an ideal gas is a function of temperature only.

c)The equation $PV^\gamma =$ constant is valid for any adiabatic process involving ideal gases.

d)The enthalpy of an ideal gas is a function of temperature only.

e)For any process the second law of thermodynamics requires that the entropy change of the system be zero or positive.

f)If the system undergoes a reversible adiabatic change, it is correct to say that the entropy of the system does not change.

g)The internal energy of an isolated system must be constant.

h)All ideal gases have the same molar heat capacity at constant pressure.

i)The Helmholtz free energy of an isolated system must be constant.

j)If a given amount of ideal gas undergoes a process during which $PV^2 = k$, where k is a constant, then it is also true that $T/P^{\frac{1}{2}} = k'$, where k' is another constant.

2. For the following multiple choice questions indicate your answers by the appropriate letters (a), (b), (c) or (d).

i) An inventor claims to have designed an engine that produces 1200 kJ of heat from a single heat reservoir while receiving 1000 kJ of heat from a single heat reservoir during the complete cycle of the engine. Such an engine would violate

(a) The first law

(b) The second law

(c) Both the first and second laws

(d) Neither the first nor the second laws

ii) A system is changed from an initial equilibrium state to the same final equilibrium state by two different processes - one reversible, and one irreversible. Which of the following is true, where ΔS refers to the system?

(a) $\Delta S_{irr} = \Delta S_{rev}$
(b) $\Delta S_{irr} > \Delta S_{rev}$
(c) $\Delta S_{irr} < \Delta S_{rev}$
(d) No decision is possible with respect to (a), (b) or (c).

iii) A hypothetical substance has the following volume expansivity and isothermal compressibility $\beta = \frac{a}{V}$, and $\kappa = \frac{b}{V}$ where a and b are constants. The equation of state of such a substance would be

(a)$V = aT + bP +$ constant
(b)$V = aT - bP +$ constant
(c)$V = bT + aP +$ constant
(d)$V = bT - aP +$ constant

iv) An exact differential expression relating thermodynamic variables is given by

$$dB = CdE - FdG + HdG$$

Which of the following would not be a new thermodynamic potential function consistent with the above expression?

(a) $B - CE$
(b) $B - HJ$
(c) $B - FG - CE$
(d) $B - HJ + FG - CE$.

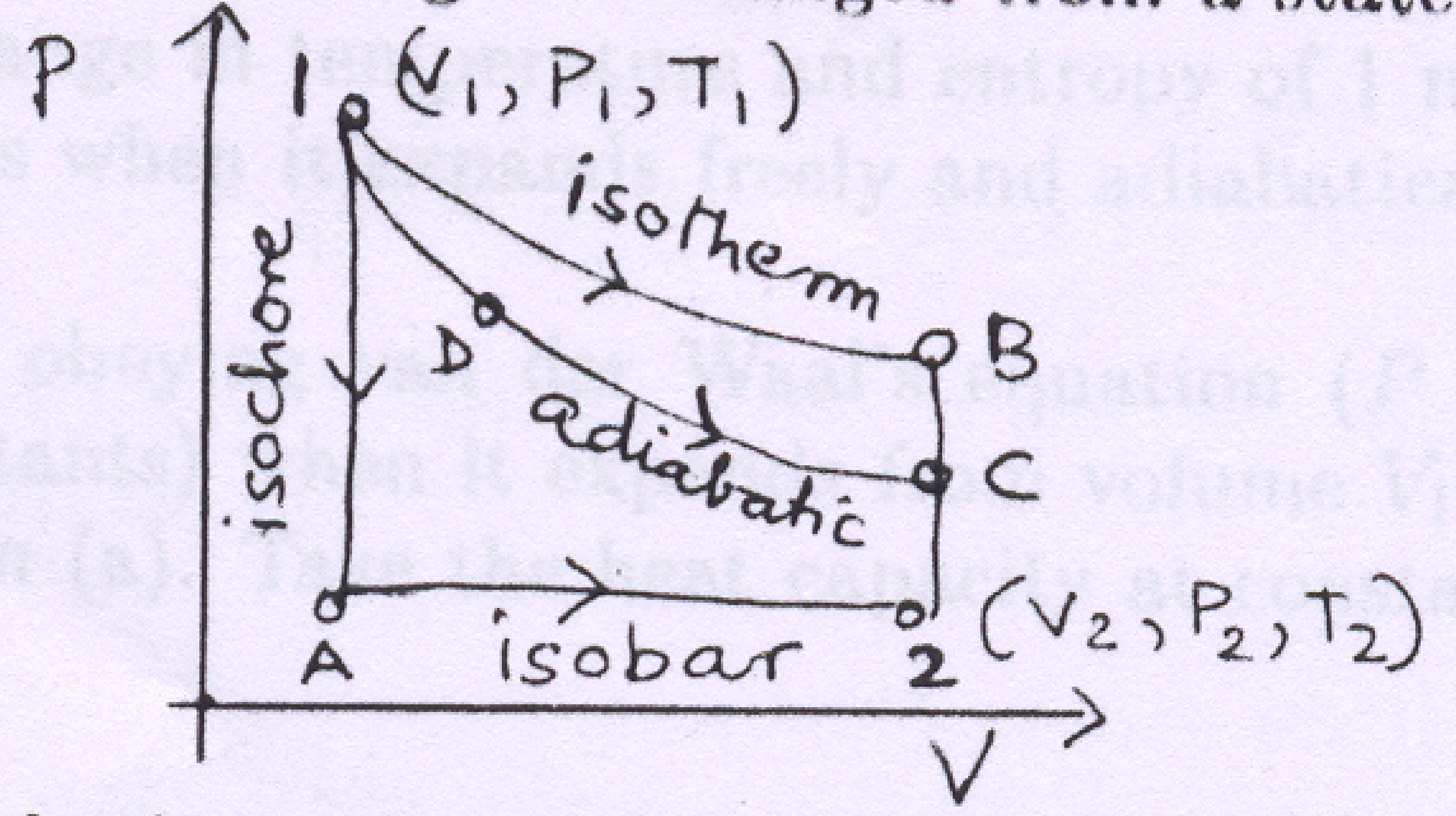
v)For a PVT system,

$$T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V$$

is *always* equal to

(a) zero
(b) $\gamma = \frac{C_P}{C_V}$
(c) R
(d) $T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P$

3. A fixed mass of ideal gas is changed from a state (P_1, V_1, T_1) to a state



(P_2, V_2, T_2) by the following three quasistatic processes as seen in the figure: (i)1A2 (ii) 1B2 and (iii) 1DC2. What is the increase in internal energy for $1 \rightarrow 2$? Also obtain the work that must be done on the system and the heat that must be added in each process. Assume that the specific heat at constant volume C_V is constant and $\frac{C_P}{C_V} = \gamma$.

4. Consider a spring which follows Hooke's Law, namely the elongation x is proportional to tension X when it is pulled at constant temperature. Determine the Helmholtz free energy F, the internal energy U and the entropy S as a function of x. Neglect the thermal expansion.

5. Einstein introduced a simplified model of solids where a solid is a collection of independent quantum harmonic oscillators of the same angular frequency ω. The energy accessible to each oscillator is given by $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2, 3 \cdots$. One mole of the solid therefore contains $3N_a$ independent one-dimensional harmonic oscillators where N_a is Avogadro's number. Using the canonical distribution show that the molar specific heat of the solid based on Einstein's model is given by:

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\frac{\theta_E}{T}}}{(e^{\frac{\theta_E}{T}} - 1)^2}$$

where $R = N_a k$ and $\frac{\hbar\omega}{kT} = \frac{\theta_E}{T}$.
Obtain the limiting forms of the specific heat for $T \gg \theta_E$ and $T \ll \theta_E$. Is the low temperature behaviour consistent with the third law of thermodynamics? Is the high temperature behaviour consistent with what is expected from the equipartition of energy theorem?

6. Find the change in temperature and entropy of 1 mole of
(a) an ideal gas when it expands freely and adiabatically from volume V_1 to V_2.
(b) a real gas obeying van der Waal's equation ($P = \frac{RT}{V-b} - \frac{a}{V^2}$ where a and b are constants) when it expands from volume V_1 to V_2 under the same conditions as in (a). Take the heat capacity at constant volume C_V to be a constant.

7. A very large mass M of porous rock is to be utilized to generate electricity by injecting water and utilizing the resulting hot steam to drive a turbine. As a result of heat extraction, the temperature of the rock drops according to $dQ_h = -MCdT$ where C is the specific heat of the rock, assumed to be temperature independent. If the plant operates at the Carnot limit, calculate the total amount W of electrical energy extractable from the rock, if the temperature of the rock was initially $T_h = T_i$, and if the plant is to be shut down when the temperature has dropped to $T_h = T_f$. Assume that the lower reservoir temperature T_l stays constant. At the end of the calculation, give a numerical value in kWh, for $M = 10^{14}$kg, $C = 1$J/g/K, $T_i = 600$ C, $T_f = 110$ C, $T_l = 20$ C. For comparison: The total electricity produced in the world in 1976 was between 1 to 2 times 10^{14} kWh.

8. Consider a system of N weakly interacting particles each of which has two energy levels, one with energy 0 and the other with energy ϵ. The system is in contact with a heat reeservoir at temperature T. Find the
(a)The average energy $\bar{E}$ of the system.
(b)The heat capacity C_V at constant volume.
(c) Show that the fluctuations in energy characterized by $\frac{\Delta E}{E}$ is small when N is large.
($\Delta E = \sqrt{< (E - \bar{E})^2 >}$).